

Full waveform inversion of shot gathers

in terms of poro-elastic parameters

De Barros (Université Joseph Fourier) &
M. Dietrich* (Institut Français du Pétrole, 1 & 4 Avenue de Bois-
Preau, 92852 Rueil Malmaison Cedex, FRANCE)

Summary

We have developed a full waveform iterative inversion procedure for determining the porosity, permeability, interstitial fluid properties and mechanical parameters of stratified porous media. The inverse problem is solved by using a generalized least-squares formalism. The proposed method achieves computational efficiency through semi-analytical, semi-numerical solutions for calculating the reference and perturbation wave fields from Biot's theory.

When this algorithm is applied to noise-free synthetic data, it is found that the inversion of a single parameter as a function of depth generally yields satisfactory results. However, simultaneous or sequential multi-parameter inversions underline, as expected, the interdependence between parameters having different physical dimensionality. The strong correlations that may exist between parameters of different types remain a major issue in multiparameter estimation, but viable solutions may be found when some a priori knowledge of the porous medium is available. For instance, the seismic data can be inverted for the saturation rate by knowing the properties of the interstitial fluid. Nevertheless, the approach proposed here militates in favor of a combination of different methods to solve the challenging task of estimating poro-elastic parameters from seismic data.

Introduction

The aim of seismic data inversion is to obtain an earth model whose response best fits the observed seismograms. Because full waveform inversion techniques are sensitive to the amplitudes and phases of the seismic disturbances, the question raises whether poro-elastic parameters can be determined from seismic signals. This question has been debated since the pioneering work of Biot (1956a, 1956b) and is of paramount importance for reservoir studies. The attempts to derive poro-elastic parameters from wave velocities and attenuation, and vice-versa (Berryman *et al.*, 2002; Pride, 2003, Pride *et al.*, 2003) obviously point to the difficulty of the task. Recently, Spikes *et al.* (2006) devised a method to interpret seismic amplitudes for lithology, porosity and pore fluid by using exhaustive Monte Carlo simulation of reservoir properties as inputs into a rock physics model.

As a contribution to this problem, we investigate here the “direct” inversion of seismic data corresponding to fluid-filled stratified porous models in terms of 8 parameters characterizing each layer. These parameters include porosity Φ and permeability k_0 , solid and fluid densities ρ_s and ρ_f , mineral modulus of the grains K_s , fluid modulus K_f , shear modulus of the grains G_s and consolidation parameter c_s (see Pride, 2003, for the definition of these parameters). Our method is based on a full waveform iterative inversion procedure carried out with a gradient technique to infer an optimum model which minimizes a misfit function. The latter is defined by a sample-to-sample comparison of the observed data \mathbf{d}_{obs} with a synthetic wavefield $\mathbf{d} = \mathbf{f}(\mathbf{m})$ in the time-space domain, and by an equivalent term describing the deviations of the current model \mathbf{m} with respect to an a priori model \mathbf{m}_0 , i.e.,

$$S(\mathbf{m}) = 1/2 \{ \|\mathbf{d} - \mathbf{d}_{\text{obs}}\|_D^2 + \|\mathbf{m} - \mathbf{m}_0\|_M^2 \},$$

where the norms $\|\cdot\|_D$ and $\|\cdot\|_M$ involve a data covariance matrix \mathbf{C}_D and an a priori model covariance matrix \mathbf{C}_M (Tarantola and Valette, 1982).

Our work is an extension to poro-elastic media of an algorithm previously developed for the inversion of plane-wave seismograms in elastic media (Kormendi and Dietrich, 1991). It is also a continuation of a recent contribution dealing with the derivation of the sensitivity operators for poro-elastic media (De Barros and Dietrich, 2006). For these reasons, we will only outline the inversion procedure and refer the reader to the publications mentioned above. We will then present inversion results obtained with noise-free synthetic seismograms for estimating the medium parameters, separately or simultaneously.

Method

The implementation of a full waveform inversion method requires several ingredients which all constitute problems to solve. Firstly, we need a forward modeling code for the geometry under consideration, i.e., a computer program capable of simulating the point source response of a layered poro-elastic medium. Secondly, when choosing a gradient technique to minimize the cost function, we need an efficient method to compute the differential or perturbation seismograms representing the sensitivity of the wave fields relative to the different model parameters. Thirdly, an inversion strategy is required for the optimization problem.

The forward modeling code has been adapted from a more complex program taking into account the coupled seismic and electromagnetic wave propagation (Garambois and Dietrich, 2002). The computation of the differential seismograms required lengthy analytical developments to establish, within the Born approximation, formulas giving the Fréchet derivatives of the solid and fluid displacements in the P - SV and SH wave cases by applying a perturbation analysis to the governing wave equations in the plane-wave domain. A total of 76 expressions involving the Green's functions between source and perturbation and between perturbation and receiver were thus obtained for the medium parameterization used (De Barros and Dietrich, 2006). Finally, we implemented the whole inversion procedure in the time-space domain by using either a conjugate gradient algorithm or a quasi-Newton method. We used in all cases diagonal covariance matrices in the model space and data space. We also imposed constraints on the model parameter values to keep them within physical boundaries. The inversion algorithm starts with an a priori model and is stopped when the cost function becomes less than a predefined minimum value or when a maximum number of iterations is reached.

Inversion results

In the following examples, we consider a vertical point force and shot gathers corresponding to vertical displacements without any noise contamination, the source-time function being perfectly known. Source and receivers are located at the free surface. We do not consider direct waves and surface waves in our computations. It should be noted, however, that our algorithm is able to handle more complex source-receiver configurations, as well as multi-component datasets.

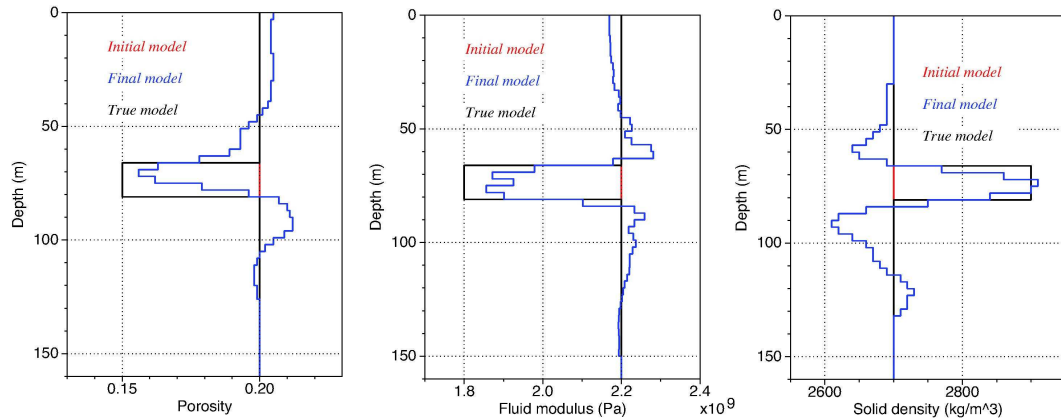


Figure 1 – Inversion results for theoretical distributions of the solid density (left), porosity (middle) and fluid modulus (right) described by “boxcar” functions. In each panel, the true model, initial model and final reconstructed model are shown in black, red and blue, respectively.

One-parameter inversion

We first assume that only one model parameter distribution is unknown, the other parameters being perfectly known and fixed. To check the performance of the inversion algorithm, we first consider a very simple variation of the model parameters in the form of a “boxcar” function. The medium is discretized with thin layers whose thickness usually represents $\frac{1}{2}$ or $\frac{1}{4}$ of the shortest wavelength. The structure is reconstructed via the inversion method described above (Figure 1). In spite of some inaccuracies, the results displayed in Figure 1 show that the inversion algorithm does a reasonable job to estimate the true models when only one parameter distribution is considered at a time. We also considered a more complex model depicted in Figure 2. Here too, we observe that the inversion procedure yields satisfactory results, although we may notice a deterioration of the results as a function of depth.

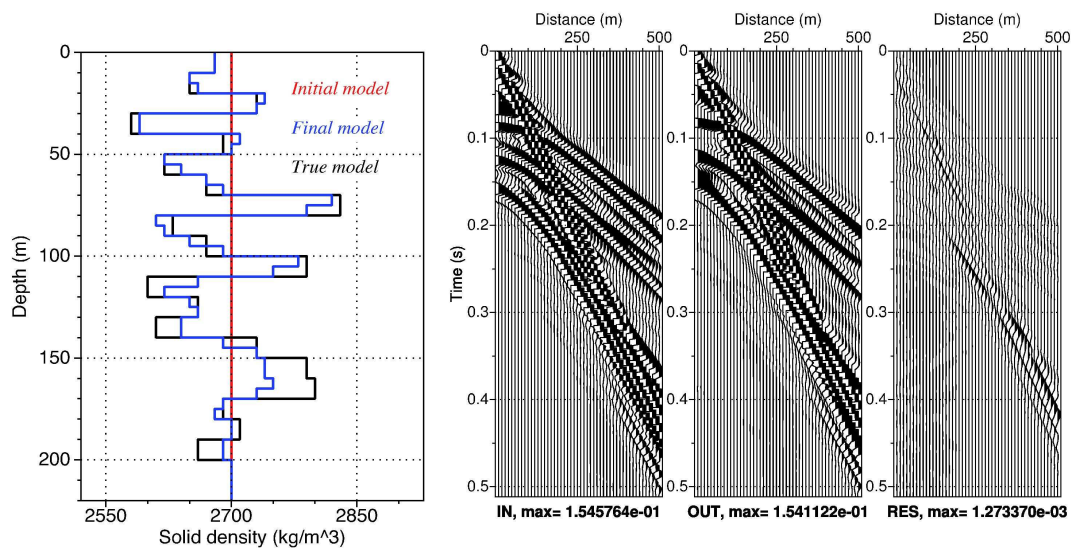


Figure 2 – Inversion of a synthetic shot gather for solid density. From left to right: panel with the true, initial and final models; seismic sections displaying the input data corresponding to the true model, the synthetic seismograms obtained at the last iteration, and the residual wave field.

Multi-parameter inversion

Next, we test our algorithm by trying to invert for several model parameters at the same time (Figure 3 below).

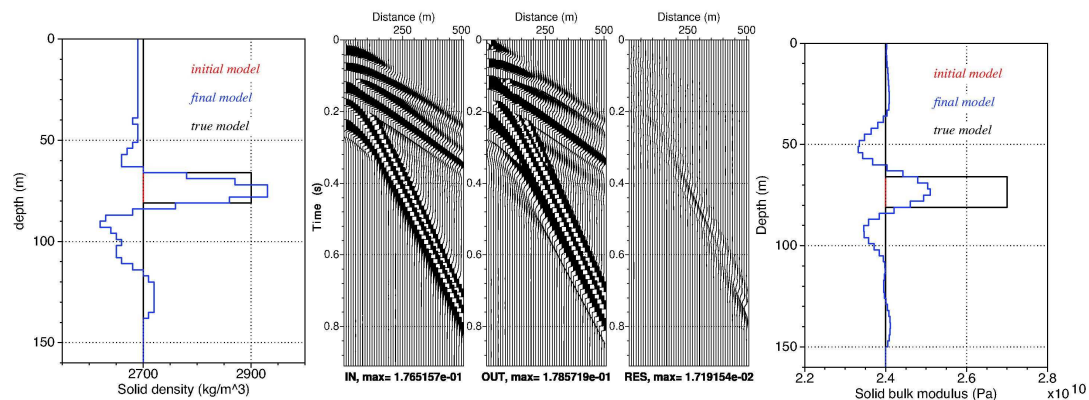


Figure 3 – Simultaneous inversion for the solid density (models on the left), and solid bulk modulus (models on the right). The seismic sections have the same meaning as in Figure 2.

This time, we notice that one of the two parameters (the solid bulk modulus) is poorly reconstructed due to parameter coupling.

Another way to proceed is to combine several parameters together and use a priori lithological information to reduce the inversion to only one parameter. An example is given below with the saturation rate chosen instead of the fluid modulus and fluid density.

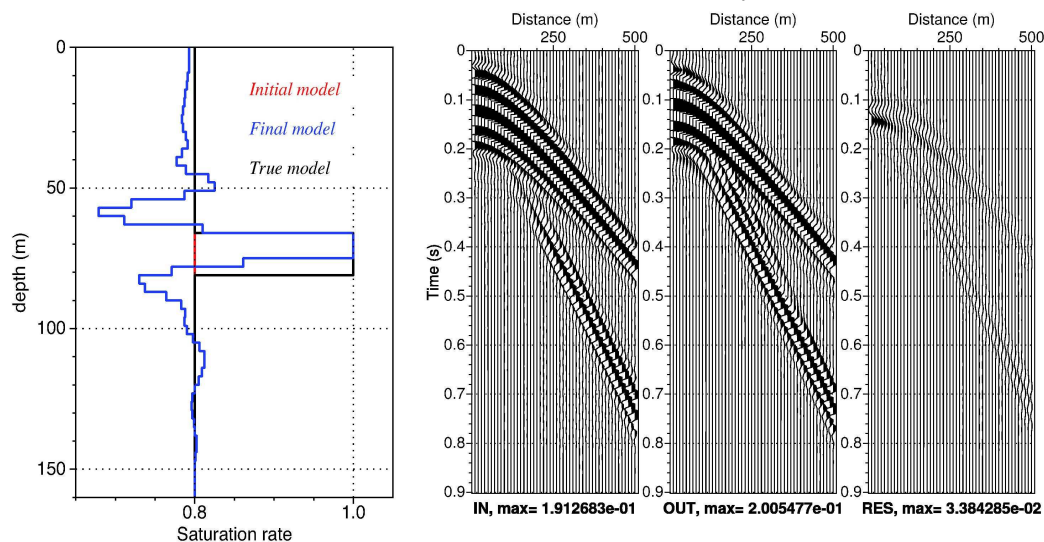


Figure 4 – Inversion for the saturation rate. Caption as in Figure 2.

Conclusions

This work represents a first attempt to directly invert seismic shot gathers in terms of the intrinsic properties of a fluid-filled porous medium. The examples considered stress the complexity of this exercise when it comes to simultaneously invert for several parameters whose perturbations have similar radiation patterns (or AVA responses). A possible way to circumvent this problem is to consider composite parameters and reliable a priori information.

Acknowledgements

We thank Bernard Valette, Stéphane Operto and Jean Virieux for many helpful discussions in the course of this work.

References

- Berryman, J. G., Berge, P. A. and Bonner, B. P. [2002] Estimating rock porosity and fluid saturation using only seismic velocities. *Geophysics*, 67, 391-404.
- Biot, M.A. [1956a] Theory of propagation of elastic waves in a fluid-saturated porous solid. I. Low-frequency range. *J. Acoust. Soc. Am.* 28, 168-178.
- Biot, M.A. [1956b] Theory of propagation of elastic waves in a fluid-saturated porous solid. II. Higher frequency range. *J. Acoust. Soc. Am.* 28, 179-191.
- De Barros, L. and Dietrich, M. [2006] First-order perturbations of the seismic response of fluid-filled stratified poro-elastic media. 76th Meeting, Society of Exploration Geophysicists, Expanded Abstracts, 2250-2253.
- Garambois, S. and Dietrich, M. [2002] Full waveform numerical simulations of seismoelectromagnetic wave conversions in fluid-saturated stratified porous media. *J. Geophys. Res.* 107, 2148-2165.
- Kormendi, F. and Dietrich, M. [1991] Nonlinear waveform inversion of plane-wave seismograms in stratified elastic media. *Geophysics* 56, 664-674.
- Pride, S. [2003] Relationships between seismic and hydrological properties. *Hydrogeophysics*. Kluwer.
- Pride, S. R., Harris, J. M., Johnson, D. L., Mateeva, A., Nihei, K., Nowack, R. L., Rector, J. W., Spetzler, H., Wu, R., Yamamoto, T., Berryman, J. G. and Fehler, M. [2003] Permeability dependence of seismic amplitudes. *The Leading Edge*, 22, 518-525.
- Spikes, K., Dvorkin, J. and Mavko, G. [2006] Rock physics model-based seismic inversion. 76th Meeting, Society of Exploration Geophysicists, Expanded Abstracts, 1645-1649.
- Tarantola, A. and Valette, B. [1982] Generalized nonlinear inverse problems solved using the least squares criterion. *Rev. Geophys. Space Phys.*, 20, 219-232.